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AN EFFICIENT FINITE ELEMENT FOR THE STUDY OF DRAG CHAINS FOR A FLOATING PIPELINE

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ABSTRACT

In areas with very uneven ocean bottom topography, the floating pipeline concept may be used. An example is the Ormen Lange gas field west of Norway. The floating pipeline has positive buoyancy and is moored to the seafloor at regular intervals by tethers. Before the tethers are installed, drag chains will keep the pipeline in position.

To study the effect of the drag chains on the pipe installation, its static configuration and its dynamic behavior, it would be very inefficient to model each drag anchor using cable elements. This would require a very high number of elements and small timesteps, while the details of the forces in the chain are not critical in the pipeline design.

Instead, a non-linear element has been developed that precisely represents the quasi-static behavior of a complete drag chain. To determine reaction forces at the top end of the chain given displacements of the top, the length of dragging chain necessary to balance the forces acting on the top is found by iteration. Proof is provided that the iteration is unconditionally stable. This iteration makes use of the classical catenary solution for the hanging part of the chain, combined with a model of the configuration of the fraction of the chain resting on the seabed. This configuration can be arbitrary, so that any combination of straight chain, heaps and coils resulting from the history of the displacement chain is accounted for. The element behaves in a coherent way also if the whole length of chain is being dragged on the seabed, lifted clear of the seabed, or landed back on the seabed.

The incremental stiffness matrix is found by numerical derivation of the reaction forces with respect to the displacements. The element is designed for use in non-linear static or non-linear time domain analyses. It has been implemented in the riser and pipeline analysis program RIFLEX and successfully tested.

INTRODUCTION

The concept of a floating pipeline tethered to the seafloor is considered for cases where the uneven ocean bottom topography makes it impossible to design a large diameter pipeline using standard methods. This concept has received particular attention in the Ormen Lange project.

At regular intervals, the floating pipeline is tethered to anchors that ensure the stability of the pipe. To ensure further stability, either between the anchors, or during the installation phase, the use of drag chains, hanging from the pipeline at intervals of several tens of meters, is considered. The lower end of the chain is left free, and comes in contact with the seafloor. The pipe finds its vertical equilibrium when the weight of the chain not in contact with the seafloor becomes equal to the buoyancy of the pipeline.

Existing finite element software, like RIFLEX, are capable of carrying out a detailed analysis of such a system. However, the analysis of a reasonable length of floating pipeline would require the modeling of a large number of drag chains, using cable elements, resulting in large finite element models with the dynamics of the pipe and the chains occuring at very different scales in both time and space. This would result in a computationaly demanding.

When studying the stresses in the pipeline, all that is needed is a reasonable approximation of the forces exerted by the chains on the pipeline, because the stress in the chain will usually not be

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critical. Further, since the eigen-periods of the chain can be expected to be significantly lower than that of the pipeline, a quasi-static model of the chain forces will be sufficient for most purposes.

The present paper discusses the theoretical basis and the implementation of a semi-analytic non-linear finite element for the drag chain. This element has only one node at the top of the chain, which can be assembled to the pipe model.

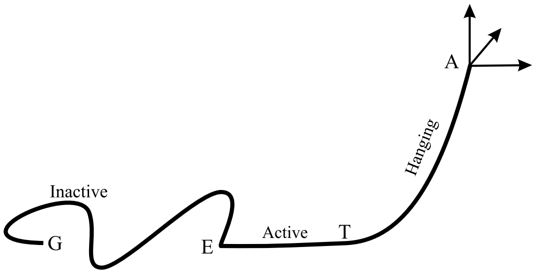

Figure 1. *Schematic view of the drag chain model*

THEORY

Introduction

In the present context, it is useful to think of a non-linear finite element as something that given the coordinates of its node(s) returns the reaction forces exerted by the element on the node(s). The expression of these forces as a function of the position of the node is exposed in the following for the drag chain element.

The forces depend not only of the position of the node, but also, on the current configuration of the chain on the seafloor, which itself depends on the history of the nodal displacement. The drag chain element will hence need to have some form of internal memory, in form of a storage of the shape of the chain on the floor. This will be discussed later on in this paper.

In order to facilitate the development of analytical expressions to describe the chain, the following assumptions are used:

1. Movements of the node are slow enough for a static solution to be valid for the chain(catenary solution).
2. Current does not affect the configuration of the chain (although a drag force at the top node A could be added separately).
3. The friction force of the chain on the seafloor is only longitudinal to the chain. This ensures that the “active” part of the chain (the part of the chain with non-zero tension) remains in a plane.
4. The slope of the seafloor is not accounted for. The depth at e.g. the lower end G of the chain is used in all the calculations for the element.
5. The chain has no axial elongation.

The catenary solution

Until explicitely stated otherwise, the chain is assumed to be within a vertical plane, and points will be described using their abscissa x and the height z above a horizontal seafloor. When current is disregarded, a static solution of the hanging part of the chain is (cf. for example Myskis (1975))

$$z(x) = r\left(\cosh\left(\frac{x-x_t}{r}\right)-1\right) \tag{1}$$

where r is the radius of curvature at the touch down point T, and x_t is the abscissa of the touch down point (cf. Figure 1). Importantly, it does not seem possible to find a closed form expression for r, given z and $x-x_t$, from (1), but we will see when considering force equilibrium that this difficulty can be circumvented.

The length along the chain from the touch down point T to a point X of coordinates abscissa x on the catenary is (from textbooks)

$$L_{TX} = r\sinh\left(\frac{x-x_t}{r}\right) \tag{2}$$

Inverting (1) gives

$$x-x_t = r\ \mathrm{acosh}\left(\frac{z}{r}+1\right) \tag{3}$$

Replacing (3) in (2) yields

$$L_{TX} = r\sqrt{\left(\frac{z}{r}+1\right)^2-1} \tag{4}$$

$$\Leftrightarrow L_{TX} = \sqrt{z^2+2zr} \tag{5}$$

$$\Leftrightarrow r = \frac{L_{TX}^2-z^2}{2z} \tag{6}$$

In particular, for $X=A$ at the top of the chain, this becomes

$$\underline{r = \frac{L_{TA}^2-z_a^2}{2z_a}} \tag{7}$$

which will be usefull in the following.

The vertical component of the force exerted by the chain on the node is equal to the submerged weight of the hanging chain

$$\underline{F_z = \omega L_{TA}} \tag{8}$$

where ω is the submerged weight per unit length of chain and L_{TA} is the length of hanging chain.

The horizontal component of the same force is such that the direction of the force is tangent to the top A of the chain:

$$F_x = \frac{F_z}{\frac{dz}{dx}(x_a)} \tag{9}$$

F_z is developped using (8) and (2) with $X = A$, while the term under the fraction bar is the derivative of (1):

$$F_x = \frac{\omega r \sinh\left(\frac{x_a - x_t}{r}\right)}{\sinh\left(\frac{x_a - x_t}{r}\right)} \tag{10}$$

$$\Leftrightarrow \underline{F_x = \omega r} \tag{11}$$

The above expressions form the base of the present element formulation. However, there is one important question they do not give the answer to: Given the position of the top node A, what is r, and where is the touch down point T? To answer that question, the friction of the chain against the seafloor must be taken into consideration.

Friction equilibrium

Consider a chain of known length L_{EA} which upper end A is dragged along a straight horizontal line until the whole chain is set in uniform translation. Where is the lower end E of the active chain (here identical to the lower end G of the chain)?

At equilibrium, the horizontal force F_x exerted by the chain on the top node A is equal to the friction F_c exerted by the seafloor on the chain:

$$F_x = F_c \tag{12}$$

This can be used to determine the radius of curvature r at the touch down point T. Using (11) for the left hand side, (12) is rewritten as

$$\omega r = \omega \mu L_{ET} \tag{13}$$

$$\Leftrightarrow \omega r = \omega \mu (L_{EA} - L_{TA}) \tag{14}$$

where μ is the Coulomb coefficient for the chain, L_{ET} is the length of dragging or active chain, and L_{TA} the length of hanging chain. r is eliminated using (7), and both sides of the equation are divided by ω, yielding:

$$\frac{L_{TA}^2 - z_a^2}{2 z_a} = \mu (L_{EA} - L_{TA}) \tag{15}$$

This is a second-degree polynomial in L_{TA}, with only one positive root:

$$L_{TA} = z_a \left(-\mu + \sqrt{\mu^2 + \frac{2\mu L_{EA}}{z_a} + 1} \right) \tag{16}$$

The variable r (which had been eliminated from the system) is then found using (7).

$$\underline{r(L_{EA}) = \frac{z_a}{2} \left(\left(-\mu + \sqrt{\mu^2 + \frac{2\mu L_{EA}}{z_a} + 1} \right)^2 - 1 \right)} \tag{17}$$

Note that $r(L)$ has positive values for $L \leq z_a$ only.

The position x_t of the touch down point T is found by inverting (1):

$$x_a - x_t = r(L_{EA}) \operatorname{acosh}\left(\frac{z_a}{r(L_{EA})} + 1 \right) \tag{18}$$

To find the lower end E of the active chain we write

$$x_t - x_e = L_{ET} \tag{19}$$

and using (13) we get

$$x_t - x_e = \frac{r(L_{EA})}{\mu} \tag{20}$$

Finally, putting (18) and (20) together, we get the abscissa of E

$$\underline{x_a - x_e = \frac{r(L_{EA})}{\mu} + r(L_{EA}) \operatorname{acosh}\left(\frac{z_a}{r(L_{EA})} + 1 \right)} \tag{21}$$

Note that the linear submerged weight of the chain ω, does not appear in (21), since all forces are proportional to it.

Equations (21) and (17) express the relation between the height of the top of the chain above the ocean floor, and the horizontal distance between the top of the chain and the lower end of the chain. This is illustrated in Figure 2.

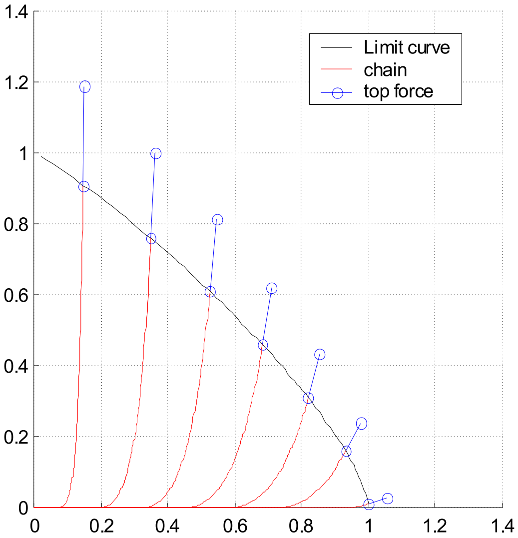

Figure 2. *Chains of unit length with* $E = G$ *at origo, with the top A placed so that the chain is just on the edge of chasing.* $\mu = 0.3$

The above relations remain valid also when E is not the lower end of the chain, but the lower end of the *active* chain. This means that there is some inactive (slack) chain below E, but the tension in the chain at E is by definition zero, so that (12) and hence (21) remain valid. In that case however, the position L_{EA} of E is no longer known. In order to determine that position, the configuration of the inactive part of the chain on the seafloor must be considered.

Chain with memory

Until now it has been convenient to assume the whole chain to be within a vertical plane. To develop a finite element problem for 3-dimensional problems, it is now time to describe all points by their coordinates x, y and z in a reference system attached to the seafloor, where z is as before the height above the seafloor. Because the top A of the chain can have followed any arbitrary trajectory, the length of chain from its lower end G to its upper end A is generally not inside a vertical plane.

To simplify the problem, it is assumed that the friction force of the chain on the seafloor is longitudinal to the chain. As a consequence of this, the length of chain between A and E is in tension with no forces normal to the vertical plane containing A and E. The chain between A and E is hence within the vertical AE plane. The length of chain between G and E, the inactive chain, is generaly not inside the vertical AE plane, and can have any arbitrary combination of loops and heaps.

The horizontal coordinates x and y of a point X along the chain are stored by the software as a function of the curvilinear abscissa L_{XA}. Using (21) and (17), the curvilinear abscissa L_{EA} of E is the value of L_{XA} that verifies

$$\begin{aligned} 0 = s(L_{XA}) = & \\ & \sqrt{(x_a - x(L_{XA}))^2 + (y_a - y(L_{XA}))^2} \\ & - \left[\frac{r(L_{XA})}{\mu} + r(L_{XA}) \operatorname{acosh}\left(\frac{z_a}{r(L_{XA})} + 1 \right) \right] \end{aligned} \tag{22}$$

In the previous sections where the chain was considered to be confined to a vertical place, the term $x_a - x_e$ expressed the horizontal distance between A and E. In three dimensions, this term in (21) must be replaced with $\sqrt{(x_a - x(L_{EA}))^2 + (y_a - y(L_{EA}))^2}$.

$s(L_{XA})$ is a decreasing function over the interval $[z_a, L_{GA}]$ where G is the lower end of the chain, so that four situations can arise when seeking a solution to $s(L_{XA}) = 0$:

1. z_a is higher than the total length of the chain: The whole chain is hanging vertically above the seafloor without touching it.
2. Any value of L_{XA} gives $s(L_{XA}) > 0$: The whole chain is dragging, and the new position of the lower end of the chain is found using (21) with $L_{EA} = L_{GA}$.
3. Any value of L_{XA} gives $s(L_{XA}) < 0$: The hanging part of the chain is vertical, and the length L_{ET} of dragging chain is zero. This situation occurs if A follows a vertical descending trajectory.
4. There is a value L_{EA} such that $s(L_{EA}) = 0$: Part of the chain is lying inactive on the seafloor while a length L_{EA} is mobilised. The chain is acting as a stable anchor.

When Equation (21) is solved numerically, $x(L)$ and $y(L)$ are found by linear interpolation between points along the chain where the position is stored. How many points are needed to give a good description of the chain's configuration on the seabed? Figure 3 shows the error on the force for a chain which top point is given cyclic displacements with increasing amplitudes. The highest amplitude is around 9m. The position of the chain is stored every 3 m. As can be seen, the error is significant when small (compared to 3m) movements of the chain are involved. Numerical studies indicate that the error decreases linearly with the number of points used. One point every 0.3m (small a distance compared to the lowest movement amplitude involved) gives a good precision.

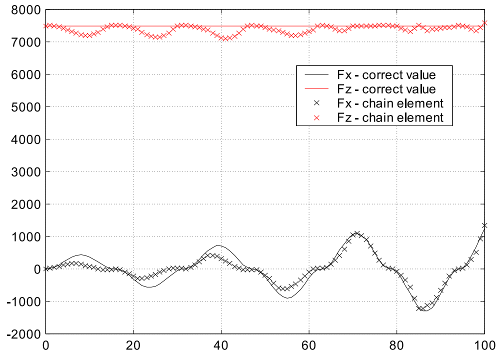

Figure 3. *Error in force prediction caused by the simplified description of the chain*

In addition to the regularly spaced points used to store the chain configuration, it is necessary to store E and use it in the interpolation, to ensure that under all circumstances, repeated computations of the top force with the same value of A give the same top force F_A.

The stored configuration of the chain on the seafloor is updated when equilibrium has been reached for a given load increment. Several schemes can be considered for this. For example the stored points on the chain can be projected on the seafloor (the vertical coordinate is discarded, the horizontal coordinates are stored without modification). An argument against this simple scheme is that in reality, if A is lowered along a vertical line, the chain will be laid in a straight, taut line until T is under A and the line is further laid in a heap at T. So in the implementation, the chain is stored as a straight, taut line between E and the projection of A on the seafloor. The remaining chain is set as a heap at the projection of A on the seafloor.

A drawback of the implemented scheme is that if A follows a downwards and sideways motion such that T remains under A, that in reality results in the chain being laid slack, then, if used with coarse steps, the update algorithm results in heaps between segments of taut chain instead of a uniformly slack chain.

Stiffness matrix

The incremental stiffness matrix is found numerically as $K = \nabla_A F_A$ (with a constant configuration of the chain on the seafloor). The forces returned in the present quasi-static algorithm depend on the coordinates of A, not on its speed or acceleration. Accordingly, the damping matrix $C = \nabla_{\dot{A}} F_A$ and mass matrix $M = \nabla_{\ddot{A}} F_A$ are both zero.

Testing

The element was developed in MATLAB before being translated to Fortran 77. This made it easy to test the behavior of the drag chain element alone when its node was submitted to complex and challenging trajectories. Figure 4 shows the configuration of the chain at the end of one of ten test trajectories that where used in the present work.

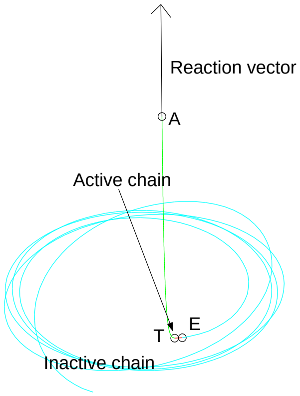

Figure 4. *Chain coiled on the seafloor, when its top A has followed a downward spiral (the number of steps and chain points used here was very large to generate a figure with a smooth configuration of inactive chain)*

Tested chain behavior include:

1. Chain loosing contact with the seafloor and landing.
2. Stable chain (there is untensionned chain on the seabed).
3. Chain dragged over the seafloor.
4. Chain laid down in a heap, then pulled aside.
5. 3-D trajectories of the top of the chain (Figure 4 is an example).
6. 2-D trajectories with “turn back”, with infinite curvature occuring at touch down point.
7. Node under the level of the seafloor (treated as node on the seafloor).
8. Stationary node - updating does not lead to change in force, thanks to the storage of E in for the interpolation scheme.

In all cases the chain configuration, the nodal force and the stiffness were monitored and found to behave satisfactorily.

Parallel to the work carried out by MARINTEK, Norsk Hydro (Finn-Gunnar Nielsen) has also investigated catenary models for drag chains. Norsk Hydro’s formulation gives the position of the node as a function of the nodal forces (which is the reverse of what is required for a finite element program). One test carried out by Norsk Hydro consisted in submitting the top of the chain to a force history representative of that of a pipeline submitted to drag from a series of waves. The resulting history of nodal displacements was communicated to MARINTEK, where it was used as input to MARINTEK's drag chain element. The resulting force history was identical to the original force history used by Norsk Hydro, provided the chain memory is refined enough. The result of the comparison for a chain memory with few points is shown in Figure 3.

After the Fortran 77 version of the algorithm was integrated into RIFLEX, several RIFLEX analyses where run successfully:

1. 200 m of floating pipeline with 11 drag chain elements attached. The pipeline is dragged sideways by prescribing displacements at its two ends. Then, current is set that pushes the pipe back in the opposite direction, resulting into a turn back situation for the drag chain elements.
2. Floating pipeline laid on uneven seabed.
3. Comparison of 80 m of floating pipeline with 1 drag chain in the middle, without seafloor contact. Displacements and force are identical to what is obtained when the chain is replaced with a vertical downward force equal to the chain's submerged weight.

Figure 5 shows the configuration of a 200m long segment of pipeline with 20 drag chains attached, floating above an uneven seabed. The finite element model of the pipeline including the drag chain has only 246 degrees of freedom. The configuration shown was reached after 25 steps, each step having converged in less than 6 Newton-Raphson iterations. This illustrates high numerical efficiency of the drag chain element.

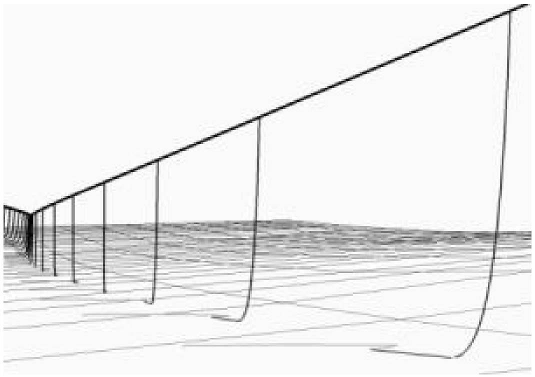

Figure 5. *Floating pipeline on uneven seabed*

CONCLUSION

Upon testing, the drag chain element proves to be very efficient and robust. As in any numerical scheme, it is important to bear in mind the assumptions discussed in this paper, which limit the element's domain of validity.

Possible future developments of the drag chain element include:

1. Accounting for current induced forces. This is of interest for computing pipe tow operations.
2. Compute a mass matrix consistent with the shape of the chain in the catenary model. The catenary solution is the solution to the static problem. In a dynamic analysis where the excitation frequencies are low compared to the eigenfrequencies of the chain, it is justifiable to use the same solution (shape function) for the chain (quasi static chain analysis).
3. Compute a damping matrix consistent with the shape of the chain in the catenary model. Compared to the mass matrix, the non-linearity of drag forces comes in as an additional difficulty.

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